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Returns to the Length of Production

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Abstract

Eugen von Böhm-Bawerk argued in his book *The Positive Theory of Capital* that longer production processes are more physically productive because the additional time allows for more capital goods to be produced which enhances the productivity of the original factors of production. He and many subsequent economists assume that there are diminishing returns to the length of the production process because the law of diminishing marginal productivity applies to the capital stock. However, the law is misapplied to the length of production because the latter violates the assumption that the units of the factor of production are homogeneous. We argue that there is a resistance to diminishing returns in the nature of the length of production due to the division of labor and the exploitation of complementarity between heterogeneous capital goods, and that this resistance may even yield increasing returns as the number of stages of production increases.

1 Introduction

The relationship between the length of a production process and its physical productivity was emphasized by Eugen von Böhm-Bawerk in his groundbreaking book *The Positive Theory of Capital* ([1889] 1959a). According to him, longer production processes allow for more capital-intensive methods of production which enhance the productivity of the original factors of production (labor and land) increasing the total physical product of given quantities of original factors. Böhm-Bawerk assumes there are *diminishing returns to the length of production* eventually set in as the process is lengthened (p. 83), i.e., that each lengthening of the production increases physical productivity by a smaller amount than the last increase, and integrates this assumption in his theory of interest in the final chapter of his book (pp. 364–365). Taussig observes that this assumption is "an essential postulate of [Böhm-Bawerk's] theory of interest" (1896, p. 313); and indeed, the formal models and attempts at improving Böhm-Bawerk's theory of interest by Wicksell ([1893] 1954, Bk. II; [1934] 1977, Pt. 2), Dorfman (1959), Lutz (1967, Chs. 1–2), and Fillieule (2015) are all based on the traditional view. Other economists such as Petr and Potuzak (2020) and Ciborowski (2023) have likewise taken the traditional view for granted. The "canonical" discussions of the structure of production found in Hayek ([1931] 1935), Rothbard ([1962] 2009, Chs. 5–9), and Garrison (2001) focus on the allocation of money across the different stages of production and only discuss the higher productivity of longer production processes in general terms, without addressing the returns to the length of production. Recent work elaborating on the physical side of the structure of production by Fillieule (2005) and Potuzak (2022, Pt. 1) continue to use diminishing returns to the length of production.

However, Taussig criticizes the traditional view as "an unduly rigid version of the direction which is likely to be followed by progress and invention" (1896, p. 313), and he writes that it is not consistent with historical counterexamples where technological innovation has increased physical returns of a production process (1908). Additionally, the law of diminishing marginal productivity does not strictly apply to the *heterogeneous* stock of capital goods produced throughout the production process; Romer (1987, 1990) writes that, given his assumptions, the inclusion of specialization in a model through a greater *variety* of factors of production can yield increasing returns to specialization. Contrary to Böhm-

Bawerk, Hayek (1937) and Lachmann (1956) write that increasing the number of stages of production may resist diminishing returns or yield increasing returns to the length of production because the series of stages that make up the structure of production allows for specialization and the exploitation of complementarity between factors of production.

In this paper, we will argue that the traditional view that there are diminishing returns to the length of production does not properly take into consideration the deepening of the division of labor and the greater exploitation of complementarity between heterogeneous capital goods that are in the nature of increasing the number of stages of production. For this reason, increasing the number of stages of production should resist diminishing returns to the length of production and might yield increasing returns. We outline the law of diminishing marginal productivity as it typically applies to homogeneous units of a factor of production in Section 2. Then we will discuss Böhm-Bawerk's theory of roundaboutness and the traditional view that there are diminishing returns to the length of production through the application of the law of diminishing marginal productivity in Section 3. We critique this application on the basis that capital goods are heterogeneous and we clarify the characteristics of two different concepts of the length of production, namely the length in terms of time and stages in Section 4. Then we argue that the nature of the length of production is such that increasing it serves to resist diminishing returns or overpower it and yield increasing returns in Section 5. Finally, in Section 6, we illustrate a more nuanced relationship between the length of production and physical output, and relate it to the structure of production framework.

2 The Law of Diminishing Marginal Product

The microeconomic analysis of production analyzes the relationship between *total product* of a physical good and homogeneous units of a physical *variable factor* (also called a variable service or variable input) in a given time period while holding the quantity of all other factors fixed. This analysis assumes that it is possible to vary the proportions of the factors being combined and that "the productive services which are held fixed in quantity must be readapted in form to the changing quantity of the variable service" and that all units of the variable factor are homogeneous (Stigler 1946, p. 122).

Additionally, there are "no improvements of the techniques of production" such as "any new inventions, improvements in organization, and the like," although it does *not* assume that it is impossible to change which technique is being used from the "*array* of possible techniques available for the production of any commodity" (Stigler 1946, p. 123). This framework is sometimes referred to as the *law of variable proportions* and is depicted in Figure 1.

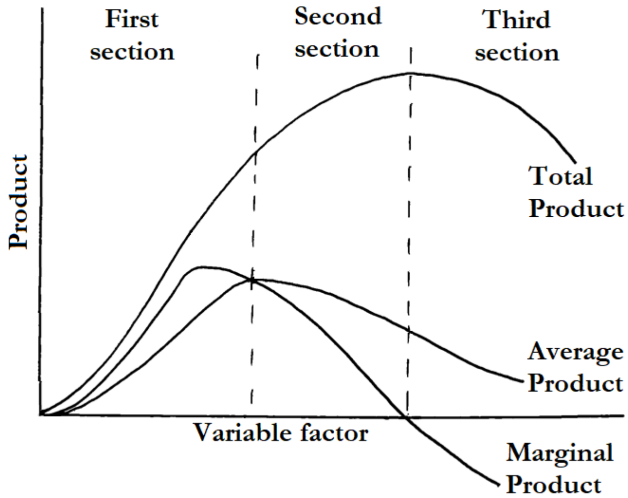

Figure 1 The law of variable proportions

The *average product* is the ratio between the total product and the quantity of the variable factors employed. The *marginal product* describes how the total physical product changes when another unit of the variable factor is added. The *law of diminishing marginal productivity* states that the marginal product will begin to decrease, i.e., that total physical product will begin to increase at a decreasing rate, beyond a certain quantity of the variable factor (in Figure 1, this takes place in the middle of the "First Section"). In assuming that units of the variable factor are homogeneous, diminishing product is not a consequence of a declining quality of the individual units of the variable factor, but because units "of equal ability are being employed less efficiently" (Stigler 1946, p. 117)

Figure 1 is divided into three "Sections" for different levels of the variable factor. In the first section, the average product is increasing and is inferior to the marginal product; diminishing marginal productivity also sets in at some point in this section. The first section ends and the second section begins when the average product and marginal product are equal, at the maximum point of the average product curve. In the second section, both the average product and the marginal product are decreasing, and the former is superior to the latter. The second section ends and the third section begins when the marginal product becomes negative, after its curve intersects with the horizontal axis. In the third section, the marginal product is decreasing and is negative. It is only wise for producers to employ a quantity of the variable factor that falls in the second section of the production function because there is no excess of either the variable factor or the fixed factors. In the third section, the marginal product of the variable factor is *negative* because there is too much of the variable factor relative to the fixed factor such that the latter is overwhelmed to the detriment of production. The first section faces the inverse problem; there is too much of the fixed factor relative to the variable factor.

Insofar as the assumptions hold, we do not dispute the law of diminishing marginal product or the law of variable proportions. The issue we will discuss in this paper involves the misapplication of these laws to cases where the assumptions do not hold.

3 The Traditional View: Diminishing Returns

3.1 The Higher Productivity of More Roundabout Methods of Production

In Böhm-Bawerk's ([1889] 1959a) *The Positive Theory of Capital*, he discusses the productivity of labor and *roundabout* methods of production, which is his terminology for process of production with longer *lengths*. The most direct method of producing consumers' goods employs only *direct labor*, which works directly on the production of consumers' goods; but these methods are limited in their productivity and in what they can produce. The use of *indirect labor* to first produce *intermediate goods* (or *capital goods*) involves "the conscription of an auxiliary power that is more potent or more clever than the human hand" which enhances the productivity of direct labor and allows for the production of *more* consumers' goods or of *better* consumers goods which could not be produced by direct labor (p. 14). Since intermediate goods themselves must be produced, the adoption of more *indirect* or *roundabout* methods of production are necessary: an increase in the length of production in terms of time (an increase in *time-length*) makes possible the addition of more stages to the production process (an increase in *stage-length*) where capital can be produced in the early stages by indirect labor and then employed to better produce consumers' goods in the later stages. Hence, Böhm-Bawerk defines "capital" as "*nothing but the sum total of intermediate products which come into existence at the individual stages of the roundabout course of progression*" (p. 14, emphasis in original). If a method of production "wisely follows an indirect [or more roundabout] course," then it "is nothing more nor less than what the economist calls *capitalist* production" because it will employ more capital to make it more physically productive than a direct (or less roundabout) method of production (p. 14).

While "the *raison d'être* of [a more roundabout] way of organising production is, of course, that by lengthening the production process we are able to obtain a greater quantity of consumers' goods out of a given quantity of original means of production" (Hayek [1931] 1935 pp. 37–38), Böhm-Bawerk writes that these more roundabout methods of production are not immediately adopted by producers because there is a "disadvantage which attends the capitalist method of production" which "consists in a sacrifice of time. Capitalist roundaboutness is productive but time consuming. It yields more or better consumption goods, but not until a later time" ([1889] 1959a, p. 82), and so more roundabout methods will only be undertaken if consumers are willing to defer their consumption for a greater satisfaction further in the future.

Rothbard ([1962] 2009) provides some clarity to some aspects of Böhm-Bawerk's theory of the relationship between roundabout methods of production, capital goods, and productivity. First, Böhm-Bawerk writes about the length of production as increasing the productivity of labor, but Rothbard generalizes his analysis writing that the productivity of all original factors of production, i.e., to both labor and land, are enhanced by longer production processes (p. 527). Second, he writes that "*man prefers his end to be achieved in the shortest possible time*" and that "the sooner any end is attained, the better" (p. 15, emphasis in original). So, Böhm-Bawerk's theory assumes that "given any present [time-length] of production, a new investment will not be in a *shorter* process because the shorter, more productive process would have been chosen first" and so "any increase in capital goods can serve only to lengthen the structure, i.e., to enable the adoption of longer and longer productive processes" (p. 538). Fisher also states this assumption clearly: he writes that the proposition that longer production processes are

> more productive than short processes is, as Böhm-Bawerk says, a general fact, not a necessary

truth. The reason lies in *selection*. It is not true that, of all *possible* productive processes, the longest are the most productive; but it is true that, of all productive processes *actually employed*, the longest are also the most productive. No one will select a long way unless it is at the same time a better way. All the long but unproductive processes are weeded out (1907, p. 353).

It is for this reason that Böhm-Bawerk ([1921] 1959b, p. 2) emphasizes that it is only a *wisely chosen* lengthening of the production process that increases productivity.

3.2 Böhm-Bawerk's Two Examples

Böhm-Bawerk ([1889] 1959a, pp. 10–11) provides two examples to illustrate the higher physical productivity that comes with longer lengths of production[1]; each example exemplified a different type of change to the production process. In his first example, he describes how a person might collect water from a spring. He may use only direct labor and collect water with cupped hands; if he has more *time*, he can engage in a different and longer method of production and first produce a *bucket* out of a log to use to collect more water; and if he has even more time, he can produce a series of *pipes* out of multiple logs to collect a greater amount of water. The water collector employs a *better type of capital good* depending on the time-length of production (i.e., how long he has to produce a capital good and employ it); he chooses to *change which capital good he will produce* in a *given number of stages of indirect labor* and employ in the stage of direct labor.

In Böhm-Bawerk's second example, he describes how a person might quarry stones. He may use only direct labor and pull out stones with his hands; if he has more time, he can first procure iron which he can then use to produce chisels and hammers to more effectively extract stones; and if he has even more time, he can use the hammer and chisel "only to drive holes into the cliffside," then "devote [his] efforts to procuring charcoal, sulphur and saltpeter and then to mixing gunpowder," and then finally "pour the powder into the holes [he] bored before, and the ensuing explosion splits the rock" (p. 11). In this example, the quarrier uses chisels and hammers in both the second and third methods of production; the longer third method introduces gun powder. The quarrier is choosing to *produce an additional*

[1] Böhm-Bawerk also provides a third example about the production of eyeglasses to illustrate how some types of consumers' goods cannot be produced with only direct labor and require the production and use of capital goods, but he does not discuss the lengthening of this production process (pp. 11–12).

capital good in *an additional stage of indirect labor* because the *time-length* of production increases sufficiently to increase the *stage-length*.

3.3 Diminishing Returns to the Length of Production

Böhm-Bawerk ([1889] 1959a) makes the argument that there should be *diminishing returns to the length of production* when the quantities of original factors are fixed. He writes of it applying specifically to the *time-length* of production: "every succeeding increase in the length of the way [of production] is marked by a concomitant augmentation in productivity," but that "each step by which the way is lengthened in marked by a proportionately smaller technical improvement" (p. 83). He qualifies this proposition writing that diminishing returns to the length of production do not necessarily set in immediately, but that "this increase [in productivity] begins to fall off *at a certain point*" (p. 85, emphasis added). Böhm-Bawerk applies the law of diminishing marginal productivity to capital as justification, writing that the law "showed that where new capital is constantly added, the most recently added installment, while it does affect an increase in the productivity of labor, does so in constantly decreasing proportion" (p. 85).[2] He applies the law to the length of production because the greater productivity of longer production processes come from employing a greater quantity of capital goods. According to Böhm-Bawerk ([1921] 1959b, p. 19), diminishing returns to the length of production can only be avoided when other factors, such as labor, increase proportionately to the capital goods accumulated over the course of the production process. In this case, the amount of capital per worker would not increase.

Wicksell ([1893] 1954, p. 122) illustrated the production function described by Böhm-Bawerk that relates total product to the time-length of production; his production function has diminishing returns at all lengths of production rather than setting in beyond a certain point. The *Wicksellian production function* is generalized in Figure 2 by the function $q = f(\tau)$, where q represents physical output

[2] Böhm-Bawerk defends diminishing returns to the length of production by arguing that it "is based on experience, and only on experience" (p. 83) and cites Thünen's description of the law of diminishing marginal productivity (p. 85). Fillieule writes that "these diminishing returns should rather be explained by the fact that there is a fixed factor, namely labor" (2019, p. 536n).

per worker and τ represents the time-length of production, and technological knowledge is constant; the function initially depicts increasing returns as τ increases, and diminishing returns once τ increases beyond $\hat{\tau}$.[3]

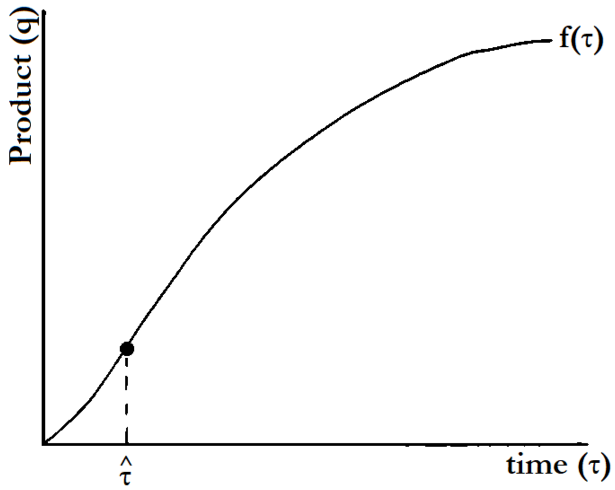

Figure 2 The Wicksellian production function

In the final chapter of *The Positive Theory of Capital*, Böhm-Bawerk puts forth his theory of interest involving a model of a producer choosing an optimal time-length of production to maximize his rate of interest ([1889] 1959a, Bk. IV, Ch. 3); although Lutz notes that it "is not the market rate of interest, but the internal rate of return (*urspriinglicher Zins*), i.e., the average rate of return earned in the process of production by the capital tied up in it" (1967, p. 12). The formal treatment of this theory by Wicksell ([1893] 1954, Bk. II) and Lutz (1967) is depicted in Figure 3, and it makes clear that diminishing returns to the time-length of production are *necessary* for the model to be solvable.[4] The model takes an amount of total wages (w) as given, which is represented by a point on the vertical axis. Then, straight lines (d) are drawn from w to the Wicksellian production function $f(\tau)$ at the different levels of τ, where the slope of these lines correspond to the rate of return. The equilibrium length of production is the one that maximizes the rate of return. In Figure 3, lengths of production τ_a and τ_b correspond to the rate of return represented by d_1; however, a higher rate of return can still be earned by choosing a length of production that falls in between τ_a and τ_b. The equilibrium length of production

[3] This more general production function is also found in Lutz (1967, p. 14).

[4] For a graphical exposition of the model, see Fillieule (2015).

is τ^* because it maximizes the rate of return, represented by d_2 which is tangent to $f(\tau)$. No lengths of production correspond to a higher rate of return; the steeper line d_3 does not correspond to any point on $f(\tau)$. For the model to be solvable, it is necessary that $f(\tau)$ have diminishing returns in the limit; otherwise, for any finite τ chosen by the producer, there would always be a longer τ with a higher rate of return.

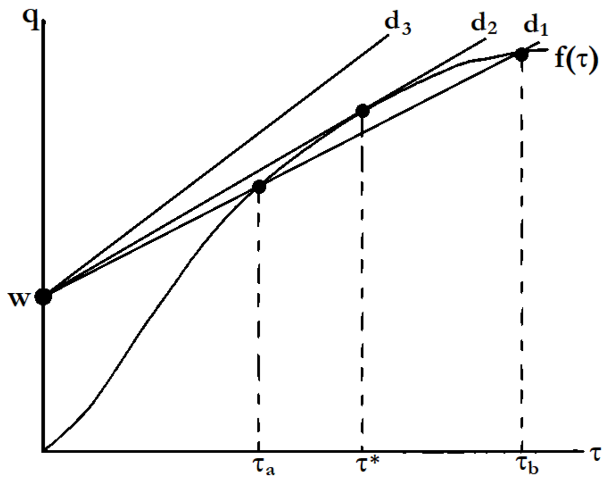

Figure 3 Equilibrium in Wicksell's model

In these models, the time-length of production is treated *as if* it were a factor of production in the context of the law of variable proportion described in Section 2, i.e., the producer only chooses time-lengths of production that fall in the "second section" of the production function in Figure 1.[5] However, the time-length of production is not in itself a factor of production, and the productivity of the original factors increases only because the time-length corresponds to the employment of a greater amount of produced factors of production.

4 Assessing the Traditional View

Appealing to the law of diminishing marginal productivity to capital to argue that there are diminishing returns to the length of production should be called into question because the law applies specifically to *homogeneous* units of a factor of production. In other words, when the time-length of a production process increases sufficiently to provide time for new capital goods to be produced, the law

[5] The extraordinary cases where the producer does not choose lengths of production that fall in the "second section" are cases where the model breaks down and yields an infinitely high or negative interest rates.

of diminishing marginal productivity strictly applies *only* in the unlikely case that the *composition of the capital goods being employed is unchanged*, i.e., if all the capital goods increase *in proportion to one another*.

In reality, it is highly likely that the composition of employed capital goods *will change*. Indeed, in both of the examples Böhm-Bawerk provides to illustrate the higher physical productivity that comes with increases to the time-length of production, increases in the length of production do not correspond to additional *homogeneous* units of a factor of production.[6] In the first example, the water collector chooses a method of production that involves a *different type* of capital good that better enhances the productivity of the original factors when the time-length of production increases sufficiently. In the second example, the quarrier chooses a more productive method that involves an *additional* capital good when the stage-length of production increases (made possible by a sufficient increase in the time-length); he can employ the additional capital good in a way that is *complementary* to the previously employed capital goods and thus further enhances the productivity of the original factors. There is also a third possibility where an increase in the stage-length allows the producer not just to add a new type of capital good, but to change the types of capital goods used in the existing stages; the producer adopts a *new combination* of capital goods that are complementary to each other. All three of these cases involve changes to the composition of capital goods being employed; new types of capital goods are being introduced, and capital goods are being employed in new combinations. Therefore, Böhm-Bawerk misapplies the law of diminishing marginal product to the length of production. It is by no means necessary that, as the length of production increases, each change in the combination of capital goods being employed has a smaller impact on productivity than the last.

The precise concept of the length of production we have in view should also be clarified. Böhm-Bawerk and Wicksell discuss returns to the length of production in terms of *time-length*; it is true that an increase in time-length on its own may allow producers to adopt better capital goods and increase

[6] Böhm-Bawerk also provides a third example about the production of eyeglasses to illustrate how some types of consumers' goods cannot be produced with only direct labor and require the production and use of capital goods, but he does not discuss the lengthening of this production process (pp. 11–12).

output, such as in Böhm-Bawerk's example about the water collector. However, Lachmann argues that our focus should instead be on the *stage-length* because "it is only if we make very restrictive assumptions that capital change can be regarded as a function of *time*," and that instead "the essence of the phenomena [of the length of production] rests in the increasing number of specific processing *stages*" (1956, p. 84, emphases added). The *process* of production is the sequence of *stages* of production; it is the stage-length that is more fundamental to the concept of the length of production. We concede that there exist cases where output increases with increases to the time-length of production even though the stage-length does not change (as we saw with Böhm-Bawerk's water collector example), but we will see in the following section that there is a *resistance to diminishing returns* in the nature of increases to the *stage-length* of production, which may even manifest in increasing returns to the stage-length of production.[7]

5 Resisting Diminishing Returns

We have seen that the standard law of diminishing marginal productivity does not strictly apply when the composition of capital goods employed in production changes, and that we should not assume that there are diminishing returns to the length of production on the basis of that law. In this section, we see that production processes with longer stage-lengths allow not only for additional capital goods to be produced, but they allow for better capital goods and for additional stages of production which serve to deepen the *division of labor* and *change the composition* of employed capital goods that make possible additional *complementarity*. These facts are not taken into account by economists such as Böhm-Bawerk who implicitly homogenize the heterogeneous capital goods employed in production, yet they bring additional increases in productivity associated with the stage-length of production *resist* the traditionally assumed diminishing returns to the length of production (to different degrees for different numbers of stages) and may yield *increasing* returns.

The stage-length of production is related to the division of labor, and increasing the stage-length

[7] We do not imply that that output increases by a smaller and smaller amount each time a producer can change from producing and employing one type of capital to a different type of capital good; these changes in productivity will not follow a precise pattern because they depend on technological knowledge.

of production deepens the division of labor. Rothbard writes that "the division of labor is not restricted to situations in which each individual makes all of one particular product," rather it "may entail the specializing by individuals in the different *stages of production* necessary to produce a particular consumers' good" ([1962] 2009, p. 102). Young (1928) argues that roundabout methods of production using indirect labor *is* the modern form of the division of labor described by Adam Smith. The greater physical productivity of the division of labor is a type of improvement that creates increasing returns or slows diminishing returns, although limited by Smith's famous theorem that "the division of labour depends upon the extent of the market" (p. 529). Young writes that "with the division of labour, a group of complex processes is transformed into a succession of simpler processes, some of which, at least, lend themselves to the use of machinery" or other capital goods that are specialized to the tasks of the laborers, and that "in the use of machinery, and adoption of indirect processes there is a further division of labour, the economies of which are again limited by the extent of the market" (p. 530). The use of specialized capital goods is limited by the extent of the market because it is often only profitable to produce these goods when the extent of the market is sufficiently large. The production of these goods is unlikely to be profitable unless they can be sold to be used by producers in many different lines of production; were this not the case, Young writes that "it would be wasteful to make a hammer to drive a single nail; it would be better to use whatever awkward implement lies conveniently at hand" (p. 530). Additionally, a larger extent of the market makes profitable large-scale production using more physically productive methods because efficiency gains "which would be uneconomical if their benefits could not be diffused over a large final product" (p. 539).

To summarize, a sufficiently great extent of the market makes the production of specialized capital goods profitable in additional stages of production profitable and facilitates the specialization of labor tasks within the individual stage. Therefore, when the stage-length of production increases, specialization contributes an additional increase to the productivity of labor, on top of the increase that comes from the amount of capital goods employed, which resists or offsetting the diminishing returns to the length of production that would apply if the composition of employed capital goods did not change.

In addition to the division of labor, the composition of capital goods employed in production changes as the length of production increases, and the *complementarity* between capital goods is in the nature of the sequential character of the production process. Hayek's (1937) discussion of chains of investments describes how complementarity relates to physical marginal productivity of capital goods in the context of the production process. He writes that "the static proposition that an increase in the quantity of capital will bring about a fall in its marginal productivity […], when taken over into economic dynamics and applied to the quantity of capital goods, may become quite definitely erroneous" (p. 174). On the contrary, depending "on the *kind of capital goods* which are produced or on the particular forms which current investment takes," the rate at which marginal productivity of capital diminishes may be lessened or the marginal productivity may *increase* (p. 174, emphasis added). Many investments are "undertaken in the expectation that further investment, for which the equipment that formed the object of the first investment will be needed, will take place at a later date" (p. 174). This *chain* of investments allows the capital goods produced by investments made earlier in the chain enhance the physical productivity of investments made later in the chain (p. 177).[8] So, increasing the stage-length of production allows different kinds of capital goods to be produced, which takes advantage of the complementarity between more factors of production and puts an upwards pressure on the marginal productivity of capital. Once again, increasing the stage-length of production offsets the traditionally assumed diminishing returns to the length of production.

Lachmann (1956, pp. 78–85) combines the insights about specialization with the insights about complementary capital goods. He writes that, "as capital accumulates there takes place a 'division of capital,' a specialization of individual capital items, which enables us to resist the law of diminishing returns" (p. 79). This division of capital is closely linked to the stage-length of production, and it is parallel to the division of labor and the complementarity of capital goods. When a production process's stage-length increases, deepening the division of labor, more specialized capital goods enhance

[8] In his paper, Hayek (1937) explicitly identifies the physical marginal productivity of capital with the rate of interest (p. 174). He holds that this rate of interest is determined by the *market for loanable funds* where the supply schedule is a function of savings and the demand schedule is a function of the marginal productivity of capital. He argues that the increase in the physical productivity of the later investments created by the earlier investments increases the demand for loanable funds, which increases the real interest rate.

productivity of labor. It does so by improving the ability of labor to perform existing tasks or make possible different types of labor tasks that are more physically productive. The specialization that comes with the division of labor within a production process resists diminishing marginal productivity of capital because new types of capital goods are produced, which allows the process to adopt new more complex "composition of capital combinations" that take advantage of complementarity between heterogeneous capital goods (p. 79). Additionally, much like how some types of labor tasks are only possible when the requisite capital goods are at the disposal of the laborer, *indivisible capital goods* require a minimum amount of complementary capital goods to be profitable because they cannot be scaled down (pp. 80–81). Lachmann emphasizes investments in producing indivisible capital goods in particular as important for resisting the diminishing marginal productivity of capital because employing them tends to completely change the method of production undertaken in the production process; their use may change the composition of the complementary capital goods employed in the production process by necessitating that capital goods be relocated and may cause other capital goods to cease to be used altogether.

The division of capital and the greater exploitation of complementarity between capital goods resulting from changes to the composition of capital are in the nature of the greater productivity of production processes with longer stage-lengths, and they are critical for resisting diminishing returns and possibly producing increasing returns to the stage-length of production increases. Lachman writes:

> Where existing capital is merely duplicated ('widened'), operated by a given labour force, diminishing returns will soon appear. Where new capital resources, but of the type employed before, are being substituted for existing labour ('deepened'), *we may have to wait a little longer for diminishing returns to make their appearance*, depending on the elasticity of substitution, but appear they will in the end. The only way in which we can hope to resist the pressure of diminishing returns is by changing the composition of capital and enlisting an indivisibility which, with fewer complementary capital resources, could not have been used. *'Higher roundabout productivity' therefore has to be interpreted in terms of this case*. The only circumstances which permit it are those circumstances which permit *a higher degree of division of capital* (p. 82, emphases added).

The stage-length of production is related to the depth capital and the complexity of the combination of capital goods that takes advantage of more complementarity. Accordingly, increasing the stage-length of production deepens capital and can resist diminishing returns to the length of production or overpower it and yield increasing returns. The latter is only possible insofar as "a sufficient number of exploitable indivisibilities" exist, and Lachmann says that they usually do exist as an empirical matter (p. 81).

Lachmann argues that the importance of the time-length of production comes primarily from making increases to the stage-length of production possible. A longer stage-length of production deepens the division of labor, allows the production of new types of capital goods, and takes advantage of complementarity between capital goods in different stages. Lachmann writes that "capital specialization as a rule takes the form of an increasing number of processing stages and a change in the composition of the raw material flow as well as of the capital combinations at each stage" (pp. 84–85), and he describes stages as "essentially layers of specialized capital equipment through which the 'original factors', i.e. raw materials, gradually filter on their journey to the consuming end" (p. 83).

5.1 Romer's Model of Increasing Returns to Specialization

Romer (1987) provides a formal model of the effect of the division of labor within a production process, where laborers specialize across a variety of activities using specialized tools and machines, which increases the variety of intermediate goods used in production. He uses this model to argue that there are increasing returns to specialization. His model uses the Dixit-Stiglitz aggregator

$$Q = L^{1-\alpha} \int_0^R x_i^\alpha \, di,$$

as a production function to take into account *different types of intermediate goods*. In this function, Q represents total annual physical output, L represents the quantity of labor factors, R represents the variety of *types* of intermediate goods produced by different labor activities, x_i measures the quantity of intermediate good type i being employed, and $0 < \alpha < 1$ is the elasticity parameter that implies that the law of diminishing marginal productivity applies to each *individual* type of intermediate good without any sections of increasing marginal returns. Romer assumes that it is possible to measure and compare units of different types of intermediate goods because they are all produced from the same "primary input” called Z (p. 57). In order to maximize physical output Q, producers allocate Z across the R individual types of intermediate goods such that all of their individual marginal products are equalized. For simplicity, Romer’s (p. 57) aggregator represents a special case where each individual type of intermediate good has the same elasticity parameter α (i.e., the exponent of x_i is α for all i) causing Z to be spread *equally* among the R different types of intermediate goods. The aggregator can therefore be written as

$$Q = L^{1-\alpha} \int_0^R \left(\frac{Z}{R}\right)^\alpha di,$$

which simplifies to

$$Q = L^{1-\alpha} R^{1-\alpha} Z^\alpha$$

in this special case.

In this model, increasing R raises Q. So, a greater variety of intermediate goods serves to resist the diminishing marginal productivity of individual types of intermediate goods because increasing R decreases Z/R, i.e., the average quantity of x_i. However, as long as Z and L are held constant, the increase in Q that comes from increasing R becomes smaller and smaller. On the other hand, increasing the average quantity of x_i by increasing Z increases total output but with diminishing marginal productivity as long as R and L are held constant.

Realistically, as the quantity of primary inputs increases, variety increases with it. It is the fact that both R and Z will *increase together* that allows Romer’s model to overcome diminishing returns. Suppose that they increase relative to each other according to the equation

$$R = bZ^\gamma.$$

where b and γ are parameters that govern the relationship between R and Z. In Romer’s aggregator, $R^{1-\alpha} Z^\alpha$ is homogeneous of degree 1 meaning that there are *constant returns* to specialization when $\gamma = 1$, there are *increasing returns* when $\gamma > 1$, and there are *diminishing returns* when $0 < \gamma < 1$. In the third case, though diminishing returns are still present, the fact that R increases still entails a resistance to diminishing returns, albeit not sufficient to overcome them. In reality, the γ parameter will not be constant as R and Z increase; there may be increasing returns at certain levels of R and (resisted) diminishing returns at other levels of R.

Romer makes some simplifying assumptions, and their conclusions should be made explicit. First, he assumes that all intermediate goods have the same elasticity parameter and consequently that marginal productivity diminishes in the same way for all intermediate goods. Second, he assumes that the intermediate goods can meaningfully be aggregated according to the quantity of an infinitely divisible "primary input" used to produce them. Were these assumptions dropped, there would not be an equal quantity of the different types of intermediate goods (i.e., $x_i = Z/R$ for all i would no longer hold) and there would not be a meaningful way to compare the quantity of the different types of capital goods; however, the marginal productivity of the individual types of intermediate goods would still be equalized by the producer. Additionally, Lachmann's (1956, p. 80) observation that changes to the composition of capital may prompt the disinvestment of certain types of capital goods is not possible in this model because of these two simplifications.

With these caveats in mind, Romer's model can serve as a basic illustration of the returns to the length of production and the idea that there may be sections of constant or increasing returns to the stage-length of production. The number of varieties of intermediate goods R can represent the stage-length of production because each stage represents a different type of labor activity that benefits from the division of labor and the complementarity of capital goods. Furthermore, the entire production process takes place in one time period where all the intermediate goods are produced simultaneously and then employed to produce the consumers' goods in Romer's model. Conceiving of production as a sequence of stages is not only a more realistic framework, but it allows for improvements in production to come from temporal complementarity, where the production that takes place in earlier stages of production increases the productivity of subsequent stages.

5.2 A Note on Aggregate Production Functions

While the standard law of diminishing marginal productivity, described in Section 2, applies only to *homogeneous* units of a factor producing a good, the neoclassical macroeconomic analysis of economic growth, based on Solow's (1956) model, applies the law to an aggregate measure of all of an economy's physical capital goods despite the heterogeneity that exists in reality. This approach uses an *aggregate production function* that relates total physical output to technological knowledge, labor, and a *composite capital factor* that represents an aggregate of "the community's *stock of capital*" (p. 66, emphasis added). The per-capita form of this function generally takes the form of

$$q_t = f(A_t, k_t),$$

where, in time period t, q_t represents per-capita total annual physical output, and k represents the per-capita quantity of the composite capital factor. This model assumes that the quantity of labor grows at a constant rate over time and that the law of diminishing marginal productivity applies to k_t, holding the other factors fixed. The variable A_t is generally identified with "technological knowledge" (Solow 1956, p. 85) but it more generally called *total factor productivity* because it represents "the portion of output not explained by the amount of inputs used in production," and so "its level is determined by how efficiently and intensely the inputs are utilized in production" (Comin 2018).

The same critiques levelled against Böhm-Bawerk's aggregation of heterogeneous capital goods in Section 4 therefore also apply to the neoclassical aggregate production function. The composite capital factor only represents the same factor if its composition remains the same, i.e., if all of the capital goods that make it up change in proportion to one another. In reality, it is likely that investing in producing new capital goods will involve *changes to the composition* of capital goods employed in production because these investments introduce new stages of production or involve different stages of production. After the investment, the per-capita quantity of the composite capital factor should instead be represented as a per-capita quantity of a *different composite capital factor*, which we can call φ_t, which entails a *different aggregate production function*, whose per-capita form will be

$$q_t = f(A_t, \varphi_t).$$

Therefore, the strict application of the law of diminishing marginal productivity of capital does not apply.

Additionally, k and φ are not comparable units because they represent different compositions of capital goods. Therefore, assuming that total factor productivity is held constant (i.e., $A_1 = A_2 = \bar{A}$), in an investment scenario where the composite capital factor changes from k to φ, output changes from $q_1 = f(\bar{A}, k_1)$ to $q_2 = f(\bar{A}, \varphi_1)$. We can only state that $q_2 > q_1$ after the investment because we are producing one type of consumers' good and because it would be unwise for a producer to make an investment that decreases output. We cannot necessarily say that $\varphi_2 > k_1$, and we cannot meaningfully plot these two production functions in a graph with a shared "capital" dimension, i.e., they cannot share the horizontal axis is Figure 1. Furthermore, Lachmann notes that the reallocation of capital goods caused by an investment in an indivisible capital good might cause some capital goods to "lose their capital character altogether" as they are ceased to be used (1956, p. 80).

The neoclassical aggregate production function assumes that the composite capital factor does not change and measures k_t by its monetary value.[9] So, using the same investment scenario with this model, output changes from $q_1 = f(A_1, k_1)$ to $q_2 = f(A_2, k_2)$, where $q_2 > q_1$ and $k_2 > k_1$; the investment is measured as part of k_t and the productive effects of changing to the composite capital factor are measured as part of the total factor productivity A_t (Comin 2018). Solow does concede that diminishing marginal productivity is not necessarily present throughout the entire function, and that "many other configurations are a priori possible" (1956, p. 71), but the standard neoclassical aggregate production function assumes that they are present throughout (Nell 2018, p. 32).

6 A New Framework

6.1 The New Wicksellian Production Function

In the previous section, we described how the productivity gains of increasing the stage-length of production may resist or overpower the diminishing returns to the length of production assumed by Böhm-Bawerk. Recognizing these facts, a realistic production function that relates the production of a

[9] Some economists have criticized measuring aggregate k_t by its monetary value because of an underlying accounting identity (Felipe and McCombie 2014).

particular consumers' good to the stage-length of the production process that produces it is therefore likely to be more complex than the function described in Section 3.3 and the Wicksellian production function illustrated in Figure 2. This *New Wicksellian production function* will be monotonically increasing because producers will not engage in a longer production process that is less physically productive. Although increasing the stage-length the production process will serve to resist diminishing returns, it does not do so to the same degree at all stage-lengths of production. It will depend on technological knowledge about possible methods of producing the consumers' good and of combining different types of capital goods, and on the feasibility and profitability of the production process. So, certain portions of the function may have decreasing, constant, or increasing returns to the length of production.

Figure 4 depicts a *hypothetical* New Wicksellian production function $q = f(\lambda)$ where q represents total annual physical output of a consumers' good per worker and λ represents the stage-length of production, and technological knowledge is constant. The returns to the stage-length of production is equal to the difference in productivity between two adjacent stage-lengths; the returns at the stage-length n is equal to $f(n) - f(n-1)$. The function's returns to the stage-length of production follow an irregular pattern because diminishing returns are resisted to different degrees at different stage-lengths. Different types of consumers' goods will have different New Wicksellian production functions with different patterns of returns to the stage-length of production, and the same type of consumers' good will have a different New Wicksellian production function if technological knowledge and capacity change.

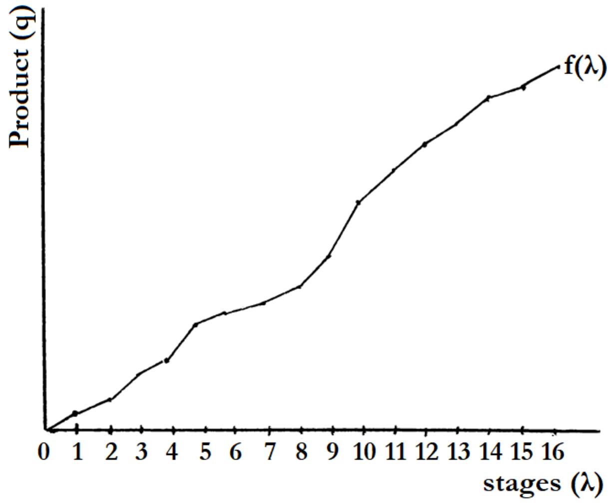

Figure 4 A hypothetical New Wicksellian production function

If the producer invests in *replicating* the existing production process, which does not increase the stage-length of production, then capital is *widened* and the composition of the capital goods involved in the process will not change, i.e., all capital goods increase in proportion to one another. In this case, the law of diminishing marginal productivity will apply *to this capital composition* because the quantity of labor and land is fixed.

It bears repeating that the horizontal axis of the function represents the length of production in terms of stages, and not identical to the length of production in terms of time (as depicted in Figure 2). Individual stages of a production process may take different amounts of time, and increasing the stage-length of production may alter how much time is taken by individual stages. Additionally, the New Wicksellian production function is different from the typical production function depicted in Figure 1. Increases in the stage-length of production do *not* correspond to increases in the amount of *homogeneous units* of a factor of production employed in production, and there are consequently additional increases to output that serve to resist diminishing returns.

6.2 Relationship with the Structure of Production

We have discussed the returns to the stage-length of production, but we have not yet discussed what determines the stage-length of production. Producers are concerned with their profits in monetary terms, and not necessarily with the physical returns to the stage-length of production. So, we will have to look at the structure of production framework found in authors such as Hayek ([1931] 1935), Rothbard ([1962] 2009, Chs. 5–9), and Garrison (2001). The structure of production is used to illustrate how money is allocated along the different stages of production. Aside from the general statement that there is a positive relationship, they leave the physical returns to the length of production unaddressed in their discussion of the monetary side of production, in part because their focus is on the monetary causes of the business cycle rather than physical production per se. The structure of production depicts a multi-stage production process that begins in the *highest* stage of production and moves to lower and lower stages until the consumers' good is produced and sold. In the highest stage of production, original factors are paid to produce a capital good which is then sold to the next stage; each subsequent stage buys a capital good from the previous stage and sells a capital good to the next stage, until the lowest stage sells a finished consumers' good is sold to a consumer. There is a price spread between each stage that reflects the interest rate in monetary terms, or the pure interest rate.

When consumers' rate of time preference decreases, they become more willing to postpone consumption and their savings rate increases, so they spend less on consumers' goods and invest more. Consequently, the monetary interest rate decreases, and it becomes profitable for producers to undertake a method of production with a longer stage-length that is more physically productive. So, it is time preference, and by consequence the monetary interest rate, that "impos[es] a limit on the length of the production processes and therefore on the maximum amount produced" of physical output (Rothbard [1962] 2009, p. 539). A decrease in the monetary interest rate, reflecting the greater willingness to postpone consumption by consumers, makes this method of production monetarily profitable to producers; its monetary rate of return was below the old interest rate but is above the new interest rate.

Given a money supply (M), consumers spend some amount of their money on consumers' goods (C), and they save and invest the remainder ($I = M - C$). The invested money is allocated to the λ stages of production, where λ_n is equal to the amount of money invested in the n^{th} stage. The savings rate ($s = I/C$) is inversely related to the monetary interest rate (i) which is equal to the price spread between the stages of production; a greater amount of savings corresponds to a decrease in the interest rate. Given C, the amount of I that is spent in λ_n is equal to $\frac{C}{(1+i)^n}$; the λ is determined by how many stages the investment I can be distributed across given i, and the total physical output can be found in the corresponding New Wicksellian production function $f(\lambda)$. Three example scenarios are depicted in Table 1. In *Scenario 1*, consumers spend $C = 201$ and save $I = 519$ with an interest rate of $i = 20\%$, and the production process is found to be $\lambda = 4$ stages long. In the other two scenarios, consumers save more, lowering the interest rate and increasing the number of stages of production.

		Stages of Production					
	C	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
Scenario 1: $I/C = 519/201$ $i = 20\%$	**201**	167	139	116	97	-	-
Scenario 2: $I/C = 555/165$ $i = 15\%$	**165**	144	125	109	95	82	-
Scenario 3: $I/C = 585/135$ $i = 10\%$	**135**	122	111	101	92	83	76

Table 1 Three structures of production where $M = 720$

The structure of production can also be depicted graphically; the upper panel of Figure 5 depicts three structures: *a*, *b*, and *c*.[10] The structures are depicted as straight lines for simplicity. The intercept on the left-hand side of the graph represents the amount of spending on the consumers' good, and the area underneath the line represents the amount of investment spending. The height of the line at different stages of production represents the amount of investment spending allocated to that stage, and the line's slope reflects the monetary interest rate. A hypothetical New Wicksellian production function is depicted in the lower panel of Figure 5. In the upper panel, *Structure a* has some level of spending on consumers' goods, investment spending, and rate of interest which determines that the length of production is equal to four stages, which corresponds to a total physical output of $f(4)$ in the lower panel. In *Structure b*, consumers spend less on consumers' goods and have a higher savings rate, the slope of the line becomes less steep reflecting the decrease in the monetary interest rate, and the length of production increase to five stages; total physical output increases to $f(5)$. Likewise for *Structure c*.

[10] The three scenarios do not correspond to the numerical values from Table 1.

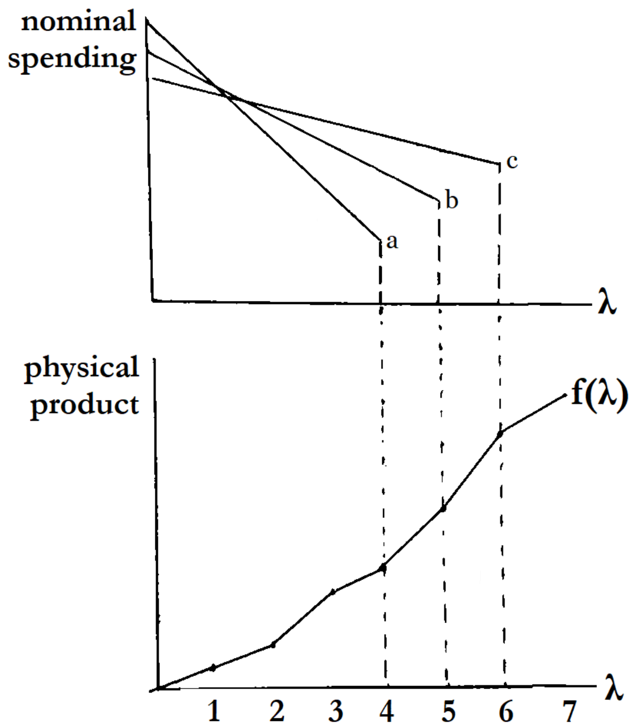

Figure 5 A structure of production with a the New Wicksellian production function

The relationship between the structure of production and New Wicksellian production function also illustrates the important distinction between the monetary rate of interest and the physical returns to capital.[11] In the structure of production, the *monetary* rate of interest (the slope of the line) *decreases each time* the stage-length of production increases after consumers' rate of savings increases. At the same time, the *physical* returns to the length of production do not necessarily decrease; the marginal productivity of capital may *increase* due to the exploitation of complementarity between capital goods that comes with an increase in the stage-length of production. Rothbard makes the same observation; he writes that "the productivity of production processes [and by consequence the marginal productivity of capital] has no basic relation to the rate of return on business investment" and that "the size of the price spread, i.e., the size of the interest rate, is determined […] by the time-preference schedules of all the individuals in the economy" ([1962] 2009, p. 424).

[11] Hayek's (1937) paper that discusses chains of investments that increase the marginal productivity of capital employs a productivity theory of interest by explicitly equating the marginal productivity of capital with the interest rate. He employs this discussion to explain why the interest rate rises near the end of a business cycle. Further research may be done in reconciling his analysis with a theory of interest that distinguishes between physical and monetary returns.

7 Conclusion and Further Research

In this paper, we have sought to describe the nature of the relationship between the length of production and physical output. The pattern of returns to the stage-length of production is not simply diminishing, rather it follows a more complex pattern because of the productivity gains that come from the division of labor and use of specialized capital goods that are complementarity to each other that resist diminishing returns. It is worth noting that the elements that resist diminishing returns and generating increasing returns, as discussed by Young (1928), Hayek (1937), and Lachmann (1956), were already present in the literature either before or during the rise in popularity of Solow's (1956) aggregate production function approach, but no serious effort was made to incorporate them. Leijonhufvud writes that the standard intergenerational equilibrium model framework assumes "smooth convexity of production sets in all dimensions," and he speculates that the reason this approach "has not managed to survive in the latter half of the twentieth century [...] is that nonconvexity is absolutely central to it. It is in essence, a translation of Smithian division of labor theory into a sequential temporal context and the *productivity of increased roundaboutness* is Smithian increasing returns in a somewhat new dress," (2006, p. 29).

The relationship between the New Wicksellian production function and the monetary interest rate must be further developed. As we saw in Section 6.2, the decrease in the monetary rate of interest makes additional methods of production become monetarily profitable to producers that are more physically production, that is, those lines of production whose monetary rate of return was below the old interest rate but is above the new interest rate. In the strictly triangular structure of production model employed by Garrison (2001), where monetary spending in the highest stage of production is equal to zero, each length of production corresponds to one rate of interest as long as the money supply remains constant. However, in the more general model without Garrison's restriction, such as the models used by Hayek ([1931] 1935) and Rothbard ([1962] 2009, Chs. 5–9), lengths of production may correspond to multiple rates of interest; this observation is made explicit by Hülsmann (2011). If the possibility that multiple interest rates are possible for the same stage-length of production is allowed, the New Wicksellian production function may shift upwards when the interest rate decreases. In other words, if there are two methods of production for the same good with the same stage-length of production, the method undertaken in an economy with a lower monetary interest rate may be more physically productive than the method undertaken in an economy with a higher monetary interest rate because only the former economy is able to use that method profitably. For example, suppose that some production processes followed the New Wicksellian production function depicted in Figure 4 as the stage-length of production was increased, but that consumers decide to stop saving and to spending more money on consumption; the stage-length of production will now begin to decrease. Now, as the stage-length falls, suppose that the monetary interest rate increases at a slower (or faster) pace than it did when the stage-length was increasing; as the stage-length falls, the quantity of physical output at each stage-length of production may be higher (or lower) than it was when the stage-length was increasing.

Further research may also examine whether equilibrium in Böhm-Bawerk and Wicksell's model of the internal rate of return, discussed in Section 3.3, can be reconciled with increasing returns to the length of production. Similar to Young's (1928, pp. 540–542) attempt to bring a model of increasing returns into equilibrium, it may be done with a dynamic model where the production function at a given point in time has diminishing returns to allow for an equilibrium, but that increasing returns manifests itself as a shift in the production function over time because of a change in a different variable.

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