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Abstract

The article proposes an explicit modelization of households behavior by describing the possible relationship between the inter-temporal substitution rate and the opportunity cost of time which could afford the missing link between consumers' choices and macro variables in an Austrian trade cycle tradition. The changes of the value of time during expansions and recessions involve direct and indirect changes of households' demand and saving which create shadow prices. The variations of shadow costs are related to the competitivity of markets restoring equilibria by means of associated changes in monetary prices.

Keywords: Inter-temporal substitution rate, originary interest, psychological interest rate, psychological time, opportunity cost of time, austrian trade cycle.

JEL: D31, J22.

Introduction

The Austrian theory of the trade cycle is based on the consequences of a monetary expansion which lowers the interest rate and provokes an over-investment (or mal-investment) in private

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business (see for instance articles by von Mises, Harberler and Garrison in Ebeling, 1978 and Hayek, 1933). As a result of greater households' income, the expansion is characterized in its last phase by a disproportionate increase in the demand for consumer goods, so that prices of consumer goods increase at the end of the expansion. In parallel, all processes of production are dilatated, which causes a disequilibrium between consumer demand and the supply of consumer goods. Generally, these models miss an explicit modelization of households behavior, since households are faced during cycles with both their monetary budget constraint and a constraint in their allocation of time: their disposable time changes inversely with their monetary resources during expansion (income increases, decreasing disposable time) and recessions (reverse changes). Alpman and Gardes (2020) supplement evidence that this implies a decrease in the opportunity cost of time of households during the great recession in the U.S. (2008-2010), which may substitute time use to monetary expenses in domestic production: for instance, households may choose less expensive transportation means while spending more time in their transportation.

It seems natural to suppose that some relationship exists between this opportunity cost of time (OCT) and the agent's time-preference of present versus future periods, which is measured by the agent's *originary interest* (the inter-temporal substitution rate (ISTR) of the literature, named also psychological interest by Allais, 1966, 1974, 2001 [1]). A change of the OCT will modify the households' allocation of income between consumption and savings (see Gardes, 2019a) which may cause some difference between market prices and the total cost of market goods: for instance, a greater opportunity cost of time increases the full cost of transportation, which implies a difference between the actual choices of the household and those which would correspond to monetary prices. A correlated change of the inter-temporal substitution rate would also modify this allocation. It seems that no model exists up to now exploring that link between the OCT and the ISTR. This article proposes a possible way to define this rela-

[1] Note that Allais, 2014 (2001 page 694, observes on historical statistics that there exists a systematic lag between the psychological interest rate and the pure interest rate of the market, the former being independent of the latter since it depends only on past changes of national global expenditure. The pure interest rate adjusts for instance to the psychological interest rate with a delay of 18 months in the British Consols during the nineteenth century. During this period, according to Allais, "'the long-term trend of the long-term pure interest rate was almost entirely governed by that of the psychological rate of interest"' (Allais, 2001, p.695)

tionship, under the assumption of a constancy of permanent full income across the household's life cycle. It then analyses the consequences of this relation over the role of households' consumption and savings in the austrian trade cycles models.

In order to estimate the opportunity of time, I use a home production model where the consumer combines time with market goods to produce activities that generate utility. Utility maximization implies that the value of time is given by the ratio of the marginal utility of time to the marginal utility of market goods. Previous papers (Gardes 2019 and 2020, Canelas et al. 2018) show that this ratio can be structurally estimated at the individual level provided that data on time and income are available. To overcome the lack of this type of data, I combine, using a statistical matching procedure described in Appendix B, a survey on household expenditures with a survey on time use made on the same year (French INSEE surveys in 2000 and surveys on expenditures and time use in the US from 2004 to 2012).

Section 1 presents the relation between the inter-temporal rate and the opportunity cost of time, section 2 its application to a model of cycle and section 3 some empirical results based on French and U.S. statistics. Section 4 concludes by a discussion of Allais's psychological time and its link with the value of time (OCT) and the implications of the model on the variations of the monetary prices during expansions and recessions. Appendix A presents the estimation of the opportunity cost of time, Appendix B the datasets and Appendix C the multipliers associated with the change in the opportunity cost of time.

I Austrian trade cycle models and the value of time

The Austrian trade cycle model is based on the effects of credit expansion by commercial banks beyond the limits of their own assets, involving the decline of interest rates below its natural level and a mal-investment (rather than over-investment) which changes the process of production according to Bohm-Bawerk's theory of the lengthening of this process during expansions and the shortening in recessions. All explanations of the business cycle which do not include these monetary factors are considered as illusory (see for instance the discussion by von Mises 1963, chapter 20.9 on 'the futile attempts to explain the cyclical fluctuations of business by a non-monetary doctrine'). This involves an inflation for capital goods followed by inflation for consumer goods as soon as the increase of the production of capital goods determines an increased distribution of income to households. The re-allocation of resources to the production of consumer goods implies losses due to unjustified investments, which give rise to a recession characterized by the shortening of the structure of production. The continuation of credit expansion would only delay that reallocation and increase its cost. The Austrian explanation implies testable empirical facts: first, the decrease of the market interest rate in the beginning of expansions; second, the price increases, first for capital goods, then for consumer goods; third, the increased incomes and expenditures of households during the expansion phase.

Austrian models discuss precisely the investment behavior of firms and the monetary channels by which money is distributed, corresponding to various financial structures characterized by endogenous money creation. I consider in this article a model analyzing households' saving and consumption choices based on the effects of income changes over their value of time. The value of time includes two different concepts: first, the opportunity cost of disposable time for home production and leisure (total time less market working time and necessary uses, such as sleeping time), which derives from the households' allocation of time among market and domestic activities according to Becker's model of the allocation of time (Becker, 1965). This opportunity cost (named ω) is generally calibrated at the level of the household's wage rate (net of all taxes and costs involved by market labor), or at a minimum wage level if the domestic production is considered as substituable to market services characterized by a low productivity and thus a cost equal to the minimum wage rate. A model of domestic production with an endogenous opportunity cost of time generalizing Becker's theory allows to estimate this opportunity cost of time at the household level (Gardes, 2019a; see Appendix A). The second dimension of the value of time refers to the inter-temporal substitution rate which governs the substitution between present and future expenditures. This inter-temporal rate ρ depends on the preference for the future which may evolve during the cycle. It generates households's saving by means of its difference with the market interest rate. It is thus particularly important to analyze the potential relationship between the two dimensions of the valuation of time by households. Theoretical considerations (see Gardes 2018) as well as empirical facts (the change of the opportunity cost during recessions, as estimated in Alpman and Gardes, 2018) indicate a positive relationship between these income changes and the opportunity cost of time. Such a co-cyclical evolution as concerns the second concept of inter-temporal substitution would help explaining the consumption and saving behavior of households during the cycle and show some potential effect of this behavior over the cyclical macro changes. The potential relationship between the two dimensions of the value of time is analyzed in the following section.

II Relating time preference to the value of time

A natural relation between ρ and ω can be extracted from their complementary relations with the household's saving rate, since the value of time impacts the household's consumption and labor income, while the rate of interest impacts its saving rate. In this first model, the household's time preference determines its saving rate, while the value of time is correlated with its expenditures, so that it could be possible to establish a relationship between them equalizing the values of its household's savings depending on these two variables. The calculus implies a positive relationship between the inter-temporal rate and the value of time. This model relies on the assumption that the decision to save part of income is taken in relation with an interest rate independently from the marginal propensity to consume, which is a heroic hypothesis whence it applies to one agent (while it is more naturally applied to aggregates in the simple keynesian model). Another way to obtain a relation between ρ and ω relies on the maximization of a utility depending both on monetary and time expenditures, but this needs specifying a particular utility and estimating its coefficients[2].

The more general method we propose relies on the assumption that the ITSR and the opportunity cost of time both influence the present value of the household's permanent full income (equal to the discounted sum of all future full incomes y^f [3]). Following the assumption made in the permanent income theory, the present value of the stream of future full incomes over the

[2] The calculus for these methods are presented in Gardes, 2021b.
[3] A pedagological model restricted to two periods is presented in Gardes, 2021b.

household's life cycle is supposed to remain constant, in spite of changes in the opportunity cost of time due to age, family structure or income (all being supposed to be anticipated by the household). This constancy is based on the assumption of identical preferences during the life cycle and the equalization of expenditures between two consecutive periods (by Hall's theorem).

This assumption supposes that the inter-temporal rate of substitution varies accordingly to the opportunity cost of time (by import or export of income between present and future periods) in order to maintain constant the total discounted value of the permanent income through the life cycle. Therefore, it is possible to calculate the change between two consecutive periods of the inter-temporal rate of substitution which corresponds to a change in the opportunity cost of time. The present value of the permanent income stream can be written for the full income flow y^f and an ISTR ρ:

$$v\left(y^f\right) = \int \left[y_t^f \exp\left(-\rho t\right)\right] dt \tag{1}$$

with the permanent full income defined by:

$$y^f = wT + y_0 = y + \omega(T - t_w) = y + \omega\sigma_i t_i. \tag{2}$$

with σ_i the elasticity of substitution between monetary and time expenditures used in the home production of commodity i (see Appendix A).

The derivative over ω of this integral equals, under usual conditions, the integral of the derivative of the discounted full income. The derivative of y^f writes, with ρ depending on ω:

$$\phi = \frac{\partial y^f}{\partial \omega} = W\frac{\partial t_W}{\partial \omega} + (T - t_W) - \omega\frac{\partial t_W}{\partial \omega} = \left(\frac{W}{\omega} - 1\right) E_{\frac{t_W}{\omega}} + \sum |\sigma_i| t_i \tag{3}$$

The integration over time (from 0 to infinity[4]) of the discounted full income $y_t^f \exp\left(-\rho t\right)$ is

[4] Households are supposed to leave their permanent wealth to their inheritants. Considering the life cycle from birth to age T multiplies the derivative of ρ over ω by the fraction $\frac{\exp(-\rho T)-1}{\rho \exp(-\rho T)}\left(T + \frac{1}{\rho}\right) - 1$ which is for instance 0.925 for $\rho = 5\%$ and T=80 years (corresponding to the average life duration in developed countries) or 0.418 for T=20 (corresponding to an evaluation at age 60 till 80).

supposed nul:

$$\int \left\{ \frac{\partial y_t^f}{\partial \omega} \right\} \exp\left(-\rho t\right) dt = \frac{\phi}{\rho} - y_t^f \frac{1}{\rho^2} \frac{\partial \rho}{\partial \omega} = 0 \tag{4}$$

Under the assumption that the household's permanent income does not change for corresponding changes of the inter-temporal rate and the opportunity cost $(d\rho, d\omega)$, the marginal propensity and the elasticity of the ITSR over the opportunity cost of time are:

$$\frac{\partial \rho}{\partial \omega} = \frac{\left(\frac{W}{\omega} - 1\right) t_W El_{\frac{t_W}{\omega}} + \sum |\sigma_i| t_i}{W t_W + V + \omega \sum |\sigma_i| t_i} \rho \tag{5}$$

$$El_{\frac{\rho}{\omega}} = \frac{(W - \omega) t_W El_{\frac{t_W}{\omega}} + \omega \sum |\sigma_i| t_i}{W t_W + V + \omega \sum |\sigma_i| t_i} \tag{6}$$

Therefore, the ISTR increases with the opportunity cost of time with an elasticity positive and smaller than 1 whence $V \geq 0$, $El_{\frac{t_W}{\omega}} \leq 1$ and $W \geq \omega$. It may vary between households, which implies different inter-temporal rates across the population, and may depend on macroeconomic conditions changing the households' wage rates, their opportunity cost of time, the allocation of time between market work and home production and the elasticity of market labor supply.

III Empirical Results on the relation between ω and ρ

The only parameter which must be estimated in equations (5) and (6) is the elasticity of market labor over the opportunity cost. Market labor time can be considered as the residual of domestic production time over the total disposable time T (supposed to be exogenous), so that this elasticity can be estimated by means of the elasticities of domestic production times t_i over the opportunity cost of time ω. Estimation of that elasticity of the market labor time using the french dataset (Gardes and Margolis, 2014) provides an estimate (depending on the elasticities of monetary expenditures over the OCT, estimated in Gardes, 2019, and on the elasticities of substitution between monetary expenditures and time uses estimated by Canelas et al., 2018) 0.861 for the whole population, with significant changes for different sub-populations, as indicated in Gardes and Margolis, Table 1. A direct estimation of the regression between $T - \sum t_i$

and ρ, reported in Table 1, gives positive but smaller estimates which are more in line with the usual estimations of the elasticity of market work over the wage rate. These estimations are used in the calculus of the marginal propensity and the elasticity of ρ over ω[5]

French dataset

On the french dataset, equation (5) gives a derivative of ρ over ω equal to 0.00221 for the whole population and an elasticity $El_{\frac{\rho}{\omega}} = 0.246$ [6]. It can be concluded that the inter-temporal rate of substitution ρ increases with the opportunity cost of time ω, with an elasticity which is smaller than one. These parameters change a lot on sub-populations : results in Table 1, in accordance with other studies (Gardes, 2018 and 2019a), show that the opportunity cost of time is greater for households having a more constrained time budget (families with children, middle aged households) and much smaller for singles. The elasticity of market work over the opportunity cost of time diminishes with age and is larger for families with children, while the elasticity of the inter-temporal rate of substitution over the opportunity cost of time increases with age and the presence of children and is one half smaller for singles compared to families with children. This shows that the effect of the value of domestic time over the inter-temporal rate of substitution differs much across the population.

The resulting change of the inter-temporal substitution rate due to income (in the cross-section dimension) is a relative increase by one third to two thirds from the 5th centile to the 95th centile of the income distribution (for instance from 3.77% to 6.23% for families with children, which corresponds to a cumulated actualized full income over the life cycle equal to 26.5 times the yearly income for 3.77% and only 16.0 for 6.23%). Therefore, the increase of yearly full income has a proportional positive direct effect on the cumulated income over the life

[5]Estimates using the higher elasticities of market work obtained by the indirect method based on domestic work in Gardes and Margolis are not so different.

[6]In this estimation, the domestic times are reduced by a factor $|\sigma_i|$ which measures the component in the domestic activity i which can be substituted to market goods and services as discussed in Appendix A. Computed on total domestic times, the corresponding $\frac{\partial \rho}{\partial \omega} = 0.00559(0.00237)$ is doubled.

TABLE 1
Derivative of ρ over ω and Elasticity

	ω	$El_{\frac{tw}{\omega}}$	$\frac{\partial\rho}{\partial\omega}$	$El_{\frac{\rho}{\omega}}$	$\rho 5^{th}(\%)$	$\rho 95^{th}(\%)$
Whole Population	6.23	0.281	0.00221	0.246	4.26	5.74
s.e.	2.02	0.017	0.00197	0.171	-	-
Singles	5.82	0.093	0.00184	0.184	4.53	5.47
s.e.	1.56	0.035	0.00145	0.126	-	-
Couples without children	6.23	0.213	0.00222	0.265	4.14	5.86
s.e.	2.36	0.027	0.00179	0.148	-	-
Couples with children	6.62	0.356	0.00279	0.338	3.77	6.23
s.e.	1.87	0.031	0.00143	0.126	-	-
Age till 35	6.02	0.413	0.00174	0.194	4.50	5.50
s.e.	1.73	0.036	0.00329	0.307	-	-
Age 36-55	6.46	0.268	0.00223	0.250	4.16	5.84
s.e.	2.30	0.019	0.00157	0.122	-	-
Age after 56	6.09	0.127	0.00321	0.358	4.04	5.86
s.e.	1.81	0.065	0.00203	0.182	-	-

Note: Bootstrap standard errors. Direct estimation of $El_{\frac{tw}{\omega}}$ on the dataset. ρ 5^{th} (respectively 95): estimate of ρ in the fifth (respectively 95^{th}) centile of the income distribution.

cycle, but also a negative indirect effect through the increase of the inter-temporal substitution rate: for instance, an increase of yearly income (over the whole life cycle) by +10% increases the ISTR (supposed at its mean 5%) by 0.114% , which implies a reduced total positive effect +7.7% instead of +10%.

U.S. dataset

During the great recession, the average opportunity cost of time decreases by 23% which implies, using the relationship between ω and ρ estimated on french data, a decrease of the ITSR by 28%. Moreover, the variation of the ITSR across the population (measured by the coefficient of variation [7]) is slightly larger during the recession period (2009-2010) by 11%, which imply similar variation of the full prices: their coefficient of variation diminishes in average by 10% between 2004 and 2008, then increases by 6% in 2009-2010, and decreases anew in 2011 and 2012 by 7%.

Work in progress: a. full prices by activity: US 2004-2012. b. Variation of monetary prices: Durables/non-durables/Services US 2000-2020

[7] ratio of the standard error over the mean

IV Discussion: Change of the Inter-temporal rate during the cycle

The changes of the ITSR during expansions and recessions follow the variation of the market interest rates, but perhaps with a larger volatility: during the U.S. great recession, the decrease of the ISTR is indeed estimated by 30% within 3 years which may differ from the corresponding variation of interest rates (see a discussion on the lags between these interests rates in Allais, 2001). This implies a change in the ratio of the households' originary rate over the market interest rate during the expansion and a decrease during the recession. Such movements will change households' total or rate of saving during expansion, which should correspond to increased consumption linked to an increased value of time (since the negative elasticity of substitution between time uses and monetary expenditures implies a positive link between the value of time and consumption: the elasticity of total expenditures relatively to ω is estimated on french data at 0.55). These changes add to those determined by increased income at the end of expansions and decreasing income during recessions. As a consequence of these changes in households' expenditures, the prices of consumer goods may increase at the end of the expansion and decrease during recessions, which probably causes an increased volatility of relative prices during that period. The change in relative prices is examined in the following section. All these changes (caused by a prior variation of the OCT) are followed by multiplicative indirect effects which are expored in Appendix C).

V Shadow prices and monetary price changes during the cycle

Rationale

Changes of the value of time determines correlated changes of full prices. These full prices are but one dimension of shadow prices corresponding to non-monetary resources (time in case of full prices), constraints (such as a liquidity constraint which add a positive shadow cost to the interest rate) or social heterogeneity. Gardes (2011) and Boelaert et al. (2017) propose to evaluate the shadow prices issued from these differences between economic agents (differing in their social locations) and discuss the inefficiency of individual choices made considering only monetary prices (known by all agents) instead of the generalized cost begetted by individual differences in the costs functions. Economic agents may indeed be fully (or partially) informed on the monetary and non-monetary components of their proper generalized costs, while other agents can be supposed to know only the corresponding monetary components, which makesthe information differ between agents.

Let us suppose that these *generalized prices* expand in periods of economic disequilibrium, for instance during the expansion when market disequilibrium are created by mal-investment and increased (and perhaps more volatile acrosss the population) households' demand, which may diminish the efficiency of the economic mechanisms tending to optimal equilibria on all markets. This increased inefficiency ends up at the end of recession when the non-monetary components of generalized prices help to the development of new market goods or services which incorporate all causes of non-monetary costs in an expansion of markets (for instance, time constraints help develop new services or durables saving households' time), so that new market goods and services come in concurrence with ancient goods. As a consequence, relative monetary prices are changing more during recessions because they incorporate part of the non-monetary (shadow) dimension of generalized prices created during the expansion phase.

The empirical test of this process can rely on the volatility of relative monetary prices for semi-aggregate expenditures. According to this assumption, the volatility of the system of relative monetary price would increase at the end of expansions and in the beginning of recessions and consequently the efficiency of market mechanism will be restored during the recession because monetary prices convey uniformally to all agents the true information on economic costs, which allows a new economic rise at the end of the recession.

Empirical consequences

Examples of shadow prices are those originating in the rising opportunity cost of time and inter-temporal rate during the expansion. During that phase, the increase of the inter-temporal rate, linked to the increase of the opportunity cost of time, provokes an increased expenditure on services and finished goods (such as prepared food or transport services) saving time. The correlated decrease in households' saving tend to increase interest rates on the market for funds in the end of the expansion while the prices of consumer goods increase. Therefore, the ratio $\frac{\rho}{r}$ may come back to 1 at the end of expansion or in recession. Rationing in consumer goods markets involves shadow prices which add to increased monetary prices of consumer goods in diminishing households' demand. The re-adjustment of the production of capital goods and consumer goods and services follows the monetary prices changes during the recession once shadow prices begin to be incorporated into monetary prices. An example of this process can be taken in the change of the time component of full prices during a recession: as the value of time diminishes during this period, the time component ωt_i of the full price for commodity i diminishes as soon as the elasticity of time use as concerns the value of time is smaller than one (in absolute value). The empirical analysis of this model is left to a future research.

Empirical test on U.S. households' expenditures data

The U.S. surveys on households' expenditures and time use have been analysed as concern the opportunity cost of time for 60 type of households and 9 years. A pseudo-panel of this dataset allows to estimate the income elasticity of the opportunity cost of time through time and to compute the corresponding full prices for 4 domestic activities: eating, housing, leisure and other activities. All full prices corresponding to these activities are characterized by negative income elasticities (respectively estimated as -0.19, -0.09, -0.26 and -0.16), which shows that they tend to decrease during expansions and increase during recessions. These variations correspond to an increased value of time during expansions, which diminishes time uses relatively to monetary expenditures and thus decreases full prices which depend on the ratio of time use over monetary expenditure.

Considering the change in monetary expenditures due to the time component of full prices (under the assumption that monetary prices do not change), the relative change in these expenditures can be calculated using the product of the income elasticities of full prices and the opportunity cost elasticities of expenditures, which are for the 4 activities: 0.11, 0.03, 0.18 and 0.10. Therefore, demand increases during expansions (and decreases in recessions) proportionally to these figures and to the rate of change of households' incomes, thus more rapidly for leisure expenditures than for housing and to a smaller proportion on food and other expenditures. This increased demand will tend to rise monetary prices for consumer goods and services in latter periods of the expansion, which compensate for the decrease of the time component of full prices and imply the stagnation of demand at the end of expansion. As a result, the variance of monetary relative prices may increase in the last periods of expansion and in recession.

An empirical analysis in progress is performed on U.S (2002-2020) individual datasets from the Consumer Expenditures and the American Time Use surveys. It will first consider the changes in full prices, then those of shadow prices revealed considering two dimensions in the dataset (Gardes et al., 2005; Gardes and Alpman, 2021). A test of a greater volatility of monetary prices during recessions compared to expansions and the decrease of the time component of full prices during the recession will complete the empirical analysis .

VI A link with Allais' psychological time

Allais proposed in a series of papers (included in Allais, 2001) to complement the calendar time (measured in days, months or years), which dates the economic events, by a *psychological time* defined by the constancy of the function of demand for money in this dimension (while the elasticity of money demand changes from one period to another in the calendar time, as observed for instance in the U.S.). Allais supposes that the psychological time is related to the *rate of expansion* which is the cumulated income changes lived by the agent in the past (with an actualization, named the instantaneous subscriptrate of forgetfulness $\chi(u)$, similar to the calculus for the flow of future incomes):

Definition of the Rate of expansion

$$Z(t) = \int \left[x(u) e^{-\int_{\tau}^{t} \chi(u) du} \right] d\tau \tag{7}$$

integrated from $-\infty$ to t.

The psychological time scale t' is defined by Allais (1966, section 2B) such that forgetfullness per unit of psychological time χ' is constant, so that it equals per period dt' in the psychological time the value of the corresponding calendar time dt:

Definition of Psychological time

$$\chi(t)dt = \chi' dt' \tag{8}$$

This psychological time can be a contraction of calendar time or an expansion of it, according to the rate of expansion, so that the ratio of these two measures of time changes from one period to another. By this relation between the psychological time and the rate of expansion, the psychological time is also related to the changes of the opportunity cost of time, which depends on present income and possibly its past variations (as well as the agent's expectation of future incomes).

Let us suppose another definition of the psychological time more in line with the meaning of the value of time: the length of calendar time Δt is supposed to be valuated at the current opportunity cost of time ω and that this value is equal to the value of the corresponding psychological time for a constant long term opportunity of time $\overline{\omega}$. This equality defines the psychological time just as it is defined by Allais in the previous equation. This new definition writes:

$$\overline{\omega} \Delta t' = \omega_t \Delta t \tag{9}$$

with ω_t the Current OCT in period t, Δ the difference between two consecutive periods in calendar time and $\overline{\omega}$ the average (long term) ω. This relation allows to estimate the current ω corresponding to the psychological time estimated by Allais's model (conditional to an estimate of the average long term ω) or, on the contrary, define the variation of the psychological time $\Delta\tau$ knowing the two opportunity cost of time. This correspondence shows that the psychological time is contracted in recessions (when the OCT diminishes) and expanded in expansions (that is $\Delta t' > \Delta t$). [8]. It is interesting to note that these variations of the psychological time could be compared to the changes of the length of production along the austrian cycle.

Conclusion

This paper provides a possible endogeneization of households behavior in a trade cycle model. Empirical analysis of correlated changes of households' opportunity cost of time and intertemporal substitution rate show that households' consumption and saving follow the path predicted by the austrian trade cycle theory. These changes modify the non-monetary components of consumption costs and may explain the loss in efficiency of market equilibria when these non-monetary costs become a large component of total costs. This loss could be estimated once the changes in virtual prices and monetary prices are estimated.

Appendix A: Estimation of the shadow price of time (Gardes, 2018, 2019a)

In order to estimate the shadow price of time, I assume that the consumer combines time with monetary expenditures to produce activities that generate utility in a model where the market working time is valued by the consumer's wage rate while the remaining time (e.g., time allocated to leisure or non-market work) is valued by a shadow price of time that may differ from the wage rate. It is assumed that the consumer's utility function is given by $u(z) = \prod_i a_i z_i^{\gamma_i}$ where a_i is a positive parameter and z_i is the quantity of the activity i produced by the combination of monetary and time inputs denoted m_i and t_i, respectively: $z_i = b_i m_i^{\alpha_i} t_i^{\beta_i}$ where $m_i = x_i p_i$

[8]Whence psychological time is extended compared to calendar time, the present takes importance compared to the future, which may decrease savings and investment in durables. This evolution is in phase with the increase of the ratio $\frac{\rho}{r}$ in the beginning of the expansion.

with x_i the quantity of the market goods i, p_i its monetary price, and b_i a positive parameter.[9] The choice of the Cobb-Douglas forms allows the parameters to be identifiable assuming that $\alpha_i + \alpha_i = 1$ (no economy of scale in the domestic productions). As we estimate the parameters locally (i.e., for each observation in the dataset), the Cobb-Douglas specifications imply simply constant substitution between time and monetary resources only in the neighborhood of each individual's equilibrium point. As discussed in Gardes (2018), the Cobb-Douglas specification allows identifying a unique shadow price of time for all activities while a more general model based on CES household production functions allows identifying activity-specific shadow prices of time together with an average value of domestic time equal to ω.

Combining the utility and the production functions allows to write the utility in terms of inputs:

$$u(m,t) = \Pi_i(a_i b_i^{\gamma_i}) \left(\prod_i m_i^{\frac{\alpha_i\gamma_i}{\sum \alpha_i\gamma_i}}\right)^{\sum \alpha_i\gamma_i} \left(\prod_i t_i^{\frac{\beta_i\gamma_i}{\sum \beta_i\gamma_i}}\right)^{\sum \beta_i\gamma_i} \tag{10}$$

$$= A\ m'^{\sum \alpha\gamma_i}\ t'^{\sum \beta_i\gamma_i} \tag{11}$$

where m' and t' are geometric weighted means of the monetary and time inputs and $A \equiv \Pi_i a_i b_i^{\gamma_i}$. In this framework, the consumer is subject to an income constraint, $\sum m_i = wt_w + V \equiv Y$, and to a time constraint, $\sum_i t_i + t_w = T$, where V is other income and t_w is the time allocated to market work. Utility maximization implies that the shadow price of time, denoted ω, is given by:

$$\omega = \frac{\frac{\partial u}{\partial t'}\frac{\partial t'}{\partial(\sum t_i)}}{\frac{\partial u}{\partial m'}\frac{\partial m'}{\partial Y}} = \frac{m'\sum \beta_i\gamma_i \frac{\partial t'}{\partial(\sum t_i)}}{t'\sum \alpha_i\gamma_i \frac{\partial m'}{\partial Y}} \tag{12}$$

The shadow price of time differs from the market wage rate when, for instance, there exists some market imperfections, transaction costs, and constraints on the labor market or in the home sector. The shadow price of time can be estimated provided that estimates of α_i, β_i, and γ_i are available, which is obtained considering the substitutions between monetary and time expenditures in the production of each final goods and by the substitution of time or money between activities (see Gardes 2019).

[9] z is assumed to depend on m (rather than x) because the dataset informs only expenditures. This approach yields consistent results when all households face the same prices.

TABLE 2
Income elasticities and parameters of the utility and domestic functions

Activity	E_{z_i/y^f}	E_{z_i/π_i}	γ_i	β_i	σ_i	$E_{z_i/\omega}$
Food	0.608	-0.850	0.138	0.675	-0.415	0.333
Housing	0.635	-1.020	0.202	0.339	-0.849	0.359
Transport	1.215	-0.746	0.151	0.543	-1.038	0.141
Clothing	0.701	-1.013	0.090	0.550	-0.287	0.649
Leisure	1.317	-0.888	0.202	0.771	-1.272	0.383
Other	1.081	-0.987	0.208	0.542	-0.651	0.413

The full income y^f is the maximum monetary income which could be earned when working during all disposable time T[10] , valued at the market wage rate net of taxes w:

$$y^f = wT + y_0 = y + \omega(T - t_w) = y + \omega\sigma_i t_i. \tag{13}$$

with σ_i the elasticity of substitution between monetary and time expenditures used in the home production of commodity i. This parameter is proved in Gardes (2021a), see subsection below, to be linked to the proportion of domestic time use which is substituable to market goods or services.

Substituable chores

Chores (domestic dayly works) are generally defined as the minimum amount of home production the value of which could be added to the value of the agent's production working on the labor market. Pure leisure activity would for instance be excluded. Home production related to food would be considered as a chore only if it corresponds to the normal amount of food for a typical similar agent, the individual surplus of a given agent being considered as linked to its particular constraints or preferences to be excluded from the chores. But this minimum amount is difficult to estimate, either directly on a survey (choosing for instance the first tercile of the distribution of time use for food among similar individuals) or by estimation of a demand

[10] Excluding sleeping time, although, as remarked by Aguiar and Hurst (2007a), sleeping time is in fact a choice variable over which individuals optimize. Sleeping time is generally excluded from domestic and leisure time in the time use literature.

system such as the Linear Expenditures System.

I propose a new method estimating the proportion of time devoted to some home activity which can be considered as a chore, by means of the degree of substitution with market alternative goods or services: the value of chores for a given domestic production (for instance of transport) is the cost to acquire a substitute on the market. The integration of that value in the market production thus proceeds from the possibility to obtain the same final good on the market at a given price which is attributed to that component of the domestic production. This requires that the elasticity of substitution between market factors of production and time must be large, so that it is possible to produce the same amount of final good with these market factors substituted to time use.

Suppose that the domestic production of some final good i is divided between 'substituable' chores 1 produced under possible substituability with market goods or services (supposed to be governed by a unitary elasticity of substitution between monetary and time inputs, corresponding to a Cobb-Douglas domestic production function: : $El_{t_{i,1}/\omega} = 1$) and non-substituable chores 2 characterized by a zero elasticity of substitution with market goods ($\sigma_{i,2} = \frac{\delta[(t_{i,2})/m_i]}{\delta(\omega)} = 0$). The same division applies to the total time devoted to the home production of the final good: $t_i = t_{i,1} + t_{i,2}$, with $t_{i,1}$ and $t_{i,2}$ the times spent in substituable (respectively non-substituable) home work producing the final good i. The elasticity of substitution of the aggregate time t_i over the opportunity cost of time ω writes, with m_i the expenditure made for market goods and services used producing the final good:

$$\sigma_i = \frac{\frac{\delta\left[(t_{i,1}+t_{i,2})/m_i\right]}{\delta(\omega)}}{\left[(t_{i,1}+t_{i,2})/m_i\right]/\omega} = \frac{\frac{\delta\left[t_{i,1}/m_i\right]}{\delta(\omega)}}{(t_{i,1}/m_i)/\omega}\frac{t_{i,1}}{(t_{i,1}+t_{i,2})} = \sigma_{i,1}\frac{t_{i,1}}{t_{i,1}+t_{i,2}} = -\frac{t_{i,1}}{t_{i,1}+t_{i,2}} = -\pi_i \quad (14)$$

The proportion of chores in this domestic activity can then be measured by the absolute value of the elasticity of substitution of the aggregate, which is smaller than one for all activities in our french dataset : 0.36 for food, 0.30 for clothing, 0.66 for leisure activities, 0.30 for

All Other activities (see Canelas et al., 2018). The total value of chores v_i in each domestic activity will be calculated by means of these elasticities of substitution and added to the monetary value of expenditures made to produce the corresponding final good: $v_i = \omega_i.t_{i,1} = |\sigma_i|.\omega_i.t_i$. A generalization to the case of multiple components of chores with different elasticities of substitution is straightforward.

The household's full resources are therefore the sum of monetary income and of that component of substituable chores valued by the estimated opportunity cost of the household:

$$y^f = Wt_w + V - \omega \sum \sigma_i t_i = Wt_w + V + \omega|\sigma|(T - t_w) \tag{15}$$

with σ the weighted average (weighted by the time budget shares) of the elasticities of substitution σ_i. The permanent income is estimated at the households' level, then aggregated over the population.

Appendix B: Datasets

The definition of comparable good and time groups of expenditure is a difficult and sometimes arbitrary operation. This rather difficult exercise needs some arbitrary assumptions about the substitution between time use and monetary expenditures (see Gronau and Hammermesh 2006 for a discussion). The commodity consumption structure does not correspond exactly to what is very often used as a standard classification even if differences can be limited. The reason is that not all time use activities can have a clear work equivalent. This is particularly the case of the leisure time. However, comparing our classification with other similar approaches (Gronau and Hamermesh, 2006) we obtain similar patterns of what these authors call ”relative goods/time intensity” defined as a ratio of good to time inputs relative to total amount of goods and time allocated to commodity production. For France, like for Israel and United States in Gronau-Hamermesh (2006) paper, the goods/time intensity is relatively high for Dwelling, Health, Clothing and to the less extent for Transport. However, the Eating item differs considerably between France and these countries, being weakly good intensive in France (0.57) and highly good intensive in the US and Israel (1.62, 1.82 respectively).

The matching procedure of a Family Expenditures survey with a Time-Use survey is either made by a regression of the time-uses observed in the matched survey using common covariates observed in the two surveys (such as education level and the age of the head, the family demographic structure or location), or using the Rubin's Multiple Imputation method (1986) which takes into account the correlations (conditional to covariates) for each variable which is matched between the two surveys (see Alpman, 2017, and Alpman and Gardes, 2015).[11] A matching of the Consumer Expenditure and the American Time use surveys using both methods shows that simple matching by regression gives often similar estimates. On the other hand, Alpman (2017) compares regression based imputation with Rubin's multiple imputation for simulated data and shows that the later furnishes imputed data which are closer to the original dataset. In Alpman and Gardes (2017), we use a survey containing both monetary expenditures and time uses for three final goods (Food, Domestic Activities and Other) and check that Rubin's procedure gives imputed values of time uses close to the true values. The french surveys used in this article have been matched by a regression.

France

The French dataset from INSEE combines at the individual level the monetary and time expenditures into a common, unique goods and services consumption structure by a statistical match of the information contained in two surveys: the Family Expenditure Survey (FES, INSEE BDF 2001) and the Family Time Budget (FTB, INSEE BDT 1999). I define 8 types of activities or time use types compatible with the available data both from FES and BDT: Eating and cooking time (FTB) and food consumption (FES), cleaning and home maintenance and dwelling expenditures (including imputed rent), clothing maintenance and clothing expenditures, education time and education expenditures, health care time and health expenditures, leisure time and leisure expenditures, transport time and transport expenditures, miscellaneous time use and miscellaneous goods and services. Time uses for all selected activities are regressed on house-

[11] Rubin's procedure does not assume implicitly that the variables to be matched are conditionally independent, which is implicitly supposed by other statistical matching methods. This assumption leads to highly biased results when it is not verified by the dataset (see Rubin, 1986 and the Stata program in Alpman, 2017).

holds' characteristics for all observation units in FTB survey and these estimations serve to predict the time spent on these activities for the corresponding units in the FES survey.

USA

The Consumer Expenditure (CE) survey is performed each quarter over 5000 to 7000 households. Usual expenditures are recorded on a weekly basis, some less frequent for one month. It is matched to the American Time Use survey (ATUS) which gives detailed information on time budgets for one year (see a presentation og these data in Aguiar et al., 2013). Both surveys are delivered each year since 2003. A continuous matching has been made for the period between 2003 and 2011 (see Alpman and Gardes, 2015) using the Rubin's method.

Appendix C: Multipliers

From: *The income multiplier and the time divider (Gardes, 2019a, Gardes et Alpman, 2020)*

Rationale

An unexpected income change dy (with the relative change $\frac{dy}{y} = \delta$) will affect all the expenditures of the household of market goods and services in proportion to their income elasticities which modifies accordingly the household total expenditure. Under the permanent income hypothesis, there is no effect of such an unexpected income change (i.e. transitory income elasticity is equal to zero) and there will not exist any cycle of causation over the opportunity cost of time and monetary expenditures or time uses. The empirical literature shows that this income elasticity is in fact positive and below one: $0 < \lambda < 1$. The proxy m' of total expenditure will therefore vary by a corresponding amount to total monetary expenditure, which finally changes the opportunity cost of time according to equation.

Secondly, this change of the opportunity cost of time will modify the ratio of monetary expenditures m_i over the time used t_i for all activities (for instance transportation), by mean of the elasticities of substitution between the two factors of the domestic production. These changes can be aggregated into the evolution between the ratio of the two proxies, m and t, for total monetary expenditures and time uses, and thus provokes a second evolution of ω by equation (4). This is the first multiplier m_1 giving rise to the final figure of the evolution $d_2\omega$ of the opportunity cost.

Thirdly, the variations of time uses t_i for all domestic productions due to the change of ω impart a corresponding inverse change of the labor supply t_w:$dt_w = -\sum t_i$ which modifies labor income in proportion of the household wage rate w (no macroeconomic effects of these micro changes being considered here). It can be supposed that the change in the market labor time is smaller than the variation in domestic times: $dt_w = -\theta \sum t_i$ with $k' < 1$. This defines a second multiplier, since the change of the households wage income will modify its consumption and indirectly the opportunity cost of time, while the opposite change in domestic time defines a time divider.

Income multiplier

$$\frac{dm'}{m'} = \left\{\sum \alpha_i \gamma_i El_{\frac{m_i}{\omega}}\right\} m_1 m_2 \delta \tag{16}$$

with $m_1 = \frac{1}{1-A}$, $m_2 = \frac{1}{1-El_{fract_w}\omega}$, A=the Elasticity of $\frac{m'}{t'}$ with respect to ω the formula of which is presented in Gardes (2019b, section 3, (iii)).

Time divider

$$\frac{dt'}{t'} = \left\{\sum \beta_i \gamma_i \left(El_{\frac{m_i}{\omega}} - \sigma_i\right)\right\} m_1 m_2 \delta \tag{17}$$

see calculus in Gardes, 2019b.

References

Aguiar, M., and Erik Hurst, Measuring Trends in Leisure: The Allocation of Time over Five Decades. *Quarterly Journal of Economics*, 122, 2007, 969−1006.

Allais, M., 1966, A Restatment of the Quantity Theory of Money. *American Economic review*, vol. 56, 5, 1123-1156.

Allais, M., 1974, The Psychological rate of Interest. *Journal of Money, Credit and Banking*, August, 285-331.

Allais, M., 2001, *Fondements de la Dynamique Monétaire*, Editions Clment Juglar.

Alpman, A., Implementing Rubin's Alternative Multiple-Imputation Method for Statistical Matching in Stata. *Stata Journal*, 16, 2016, 717−739.

Alpman, A., Gardes, F., Thiombiano, N. (2017), The pertinence of matching surveys on households time use and expenditures: a test on real data, *w.p. CES 2017*, Paris School of Economics, University Paris I Panthéon Sorbonne.

Becker, G. (1965), A Theory of the Allocation of Time. *The Economic Journal*, 75, 1965, 493−517.

Boelaert, J., Gardes, F., Langlois, S. (2017), Convergence des Consommations entre Classes Socioéconomiques et Contraintes Non Monétaires au Canada. *L'Actualité Economique*, Vol. 93, 4, December 2017.

Canelas, Carla , Gardes, François, Merrigan, Philip and Salazar, Silvia (2018), Are Time and Money Equally Substitutable for All Commodity Groups in the Households Domestic Production? *Review of Economics of the Household.*

Ebeling, R.M., The Austrian Theory of the Trade Cycle. *Center for Libertarian Studies*, 1978; Ludwig von Mises Institute, 1996.

Gardes, F. (2011),Différenciation Sociale et Dynamique des Prix dans une Economie Walrassienne, in *Baranzini, R., A. Legris et L. Ragni (ds): Léon Walras et léquilibre économique général, Recherches récentes*, Paris, Editions Economica, pp. 291-307.

Gardes, F. (2018), On the value of time and human life, *Cahiers du Centre d'Economie de la Sorbonne 2018.28*, University Paris I Panthéon Sorbonne.

Gardes, F. (2019a), The Estimation of Price Elasticities and the Value of Time in a Domestic Production Framework: an Application Using French Micro-Data. *Annals of Economics and Statistics*, 135, September, 69-100.

Gardes, F. (2019b), Income Multiplier and Time Divider as Possible Microeconomic Foundations of the Okuns Law *W.p. Un. Paris I*, July.

Gardes, F. (2020a), A solution to the estimation of an elarged GDP including domestic production: an estimation on individual data, *Cahiers du Centre d'Economie de la Sorbonne*, University Paris I Panthéon Sorbonne.

Gardes, F. (2020b), Infering time Preference from the Value of Time, *Cahiers du Centre* *d'Economie de la Sorbonne*, University Paris I Panthéon Sorbonne.

Gardes, F. (2020c), Biases on Variances estimated on Large Datasets, *Cahiers du Centre d'Economie de la Sorbonne*, University Paris I Panthéon Sorbonne.

Gardes, F., Alpman, A. (2020), Il est Temps dAnalyser la Loi dOkun et le Bien-łtre des Mnages en Termes de Production Domestique, *Revue Economique*, vol. 71, n6.

Gardes, F. and Alpman, A., 2021, La Théorie de la Production Domestique et la Résolution du Biais d'Endognéité des Elasticités Revenu Estimées sur Données Transversales, *Revue Economique*, vol. 72, n6.

Herbener, M. (2011), *The Pure Time-Preference Theory of Interest*, Ludwig von Mises Institute.

Gronau R and D.S. Hamermesh (2006), Time vs. Goods: the Value of Measuring Production Technologies, Review of Income and Wealth, 52.1 (2006): 1-16.

Sunstein, C.R. (2014) *Valuing Life*. University of Chicago Press.

von Mises, L. (1963) *Human Action: A Treatise on Economics*.Liberty Funds, Indianapolis.